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USE OF A BEM MODEL TO SIMULATE UNSTEADY COUPLED HEAT AND MOISTURE FLOW THROUGH POROUS SOLID

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Abstract. The problem of unsteady coupled moisture and heat energy transport through porous solid is studied numerically using singular boundary integral representation of the corresponding governing equations. Bench mark example moisture uptake in a semi-infinite region is studied numerically.

1 INTRODUCTION

Building materials are in general porous, composed of solid matrices and pores. Within the pores, moisture can exist in any of three thermodynamic states of matter, i.e. vapor, liquid, and solid [1, 2]. However, moisture transport is possible only in the case of gaseous/vapor and liquid states. The main moisture transport mechanisms are vapor diffusion, capillary suction or a combination of both, depending on the moisture content of the material.

In the paper, the numerical model based on boundary element method (BEM) is considered to solve coupled nonlinear heat energy and moisture transport through porous media [8]. Since the singular integral representation is based on the use of an appropriate fundamental solution which incorporates more or less physics of the transport phenomenon such as accomulation and diffusion of the field function, the accurate description of different time and length scales can be accomodated and treated much more accurately in a physically and mathematically justified manner. One serious drawback of the BEM is

that it leads to a fully populated system of equations. However, this can be overcome efficiently by the sub-domain or macro element approach, yielding a sparse system similar to FEM and FDM schemes [7].

The first part of the present work describes the problem. Relevant governing energy and moisture transport differential equations are considered [1, 2]. Next, the corresponding singular integral representation is developed describing nonlinear heat and moisture diffusion in an integral form. The nonlinearity of the coupled diffusion problem is accomplished using an iterative solution strategy, based on an under-relaxation procedure. One bench mark example, e.g. moisture uptake in a semi-infinite region, is given to demonstrate the efficiency and accuracy of the proposed solution strategy [2, 3].

2 Governing equations for two-phase system

Let us consider a two-phase thermodynamic system in a control volume Ω bounded by a control surface Γ, where the indices l and v represent the two phases. For example, l may refer to the liquid water and v to the vapor water in a liquid/vapor moisture system.

2.1 Moisture transport equation

The mass balance equation describing accumulation within the control volume, mass flux in and out of the control volume and generation of a species via reaction/phase change written for the water vapor and liquid water [1, 2]

$$\rho_m \frac{\partial Y_v}{\partial t} = -\vec{\nabla} \cdot \vec{j}_v - \dot{m}_C \qquad \text{and} \qquad \rho_m \frac{\partial Y_l}{\partial t} = -\vec{\nabla} \cdot \vec{j}_l + \dot{m}_C, \tag{1}$$

where the dimensionless field functions $Y_v = m_v/m_m$ and $Y_l = m_l/m_m$ represent mass fraction or moisture ratio of water vapour and liquid water, respectively, and m_m represents mass of the dry porous material. The vector quantities $\vec{j}_v$ and $\vec{j}_l$ denote the nonconvective water vapour and liquid water mass fluxes, respectively, and $\dot{m}_C$ represents moisture condensation/evaporation mass rate. The basic governing conservation equation for the moisture flow through a porous solid can now be derived by adding the individual species conservation eqs.(1), yielding

$$\frac{\partial W}{\partial t} = -\vec{\nabla} \cdot \vec{j} = -\vec{\nabla} \cdot \vec{j}_v - \vec{\nabla} \cdot \vec{j}_l, \tag{2}$$

where the derived potential field function $W = \rho_m Y_v + \rho_m Y_l = m_w/V_m$ represents moisture content $m_w = m_v + m_l$ per volume of dry material V_m. The model is based on the assumption that the water transport can be divided into vapor and liquid flows [1].

Using Fick and Darcy constitutive models for expressing the vapour diffusion mass flux $\vec{j}_v$ and liquid conduction mass flux $\vec{j}_l$ due to capillary suction and gravity effect, respectively, one can write

$$\vec{j}_v = -\delta_p(W,T)\vec{\nabla} p_v \qquad \text{and} \qquad \vec{j}_l = -D_l(W,T)\vec{\nabla} p_l + D_l(W,T)\rho_l \vec{g}, \tag{3}$$

where the transport coefficients δ_p and D_l are vapour permeability and liquid conductivity of the material, respectively, and the quantity $\vec{g}$ is the gravity acceleration. The transport coefficient D_l is given as follows

$$D_l(W,T) = \frac{k_l(W)\rho_l}{\eta_l(T)}, \tag{4}$$

where k_l is the permeability of the material, η_l and ρ_l are the dynamic viscosity and mass density of the liquid water, respectively. The liquid flux $\vec{j_l}$ can be rewritten in the form for the suction pressure, $p_{suc} = p_a - p_l$, where p_a is the atmospheric pressure, yielding

$$\vec{j_l} = D_l \vec{\nabla} p_{suc} + D_l \rho_l \vec{g}, \tag{5}$$

where $p_{suc} = p_{suc}[T, p_v, p_s(T)]$ and p_s is the saturation vapour pressure.

One may apply moisture content W for the driving potential and the relation for the water retention curve $W = W(p_l)$. The moisture content gradient can be written as

$$\vec{\nabla} W = \frac{dW}{dp_l}\vec{\nabla} p_l \qquad \text{or} \qquad \vec{\nabla} p_l = \frac{1}{dW/dp_l}\vec{\nabla} W, \tag{6}$$

where dW/dp_l is the slope of the water retention curve, yielding

$$\vec{j_l} = -D_W \vec{\nabla} W + D_l \rho_l \vec{g} \qquad \text{and} \qquad D_W = \frac{D_l}{dW/dp_l} = \frac{k_l \rho_l}{\eta_l dW/dp_l}. \tag{7}$$

The eq.(5) or eq.(7) can be rewritten for the p_v and T field functions. The vapour/liquid water two-phase system is in equlibrium when the pore water pressure p_l or the suction pressure p_{suc} and the relative humidity φ satisfies the Kelvin moisture state relation [1]

$$p_l \approx \frac{RT\rho_l}{M_w}\ln(\varphi) \qquad \text{or} \qquad p_{suc} \approx p_a - \frac{RT\rho_l}{M_w}\ln(\varphi) \tag{8}$$

with M_w, R and p_a being the water molecular mass, the universal gas constant and atmospheric pressure, respectively. Thus, an expression for suction pressure gradient as a function of temperature T and vapour pressure p_v can be obtained by making use of the partial differentiation of eq.(8) as follows

$$\begin{aligned}\vec{\nabla} p_{suc} = -\vec{\nabla}\left(\frac{RT\rho_l}{M_w}\ln\left(\frac{p_v}{p_s}\right)\right) &= -\frac{R\rho_l}{M_w}\ln(\varphi)\vec{\nabla}T + \frac{RT\rho_l}{M_w}\frac{1}{p_s}\frac{dp_s}{dT}\vec{\nabla}T - \frac{RT\rho_l}{M_w}\frac{1}{p_v}\vec{\nabla}p_v \\ &= -\frac{R\rho_l}{M_w}\left[\left(\ln(\varphi) - \frac{T}{p_s}\frac{dp_s}{dT}\right)\vec{\nabla}T + \frac{T}{p_v}\vec{\nabla}p_v\right],\end{aligned} \tag{9}$$

resulting in the following form of the constitutive model

$$\vec{j_l} = -D_l\frac{R\rho_l}{M_w}\left[\left(\ln(\varphi) - \frac{T}{p_s}\frac{dp_s}{dT}\right)\vec{\nabla}T + \frac{T}{p_v}\vec{\nabla}p_v\right] + D_l\rho_l\vec{g}. \tag{10}$$

Substituting mass flux eqs.(5) into conservation eq.(2) yields the governing moisture transport equation due to water vapour diffusion, liquid water conduction and gravity

$$\frac{\partial W}{\partial t} = \vec{\nabla} \cdot \left(\delta_p \vec{\nabla} p_v \right) - \vec{\nabla} \cdot \left(D_l \vec{\nabla} p_{suc} \right) - \vec{\nabla} \cdot \left(D_l \rho_l \vec{g} \right). \tag{11}$$

The moisture transport eq.(11) is comprised of various moisture driving potentials, e.g. moisture content W, partial water vapour pressure p_v and the suction pressure p_{suc}. These driving potentials can be expressed in terms of a single transport potential. Let us first formulate the moisture transport eq.(11) for the driving potentials relative humidity φ and temperature T [2]. The chosen potentials are continuous field functions at the interface of two layers of materials having different moisture storage properties (sorption and retention), therefore they are continuous field functions throughout the solution domain. Consequently, all terms in eq.(11) have to be mathematically transformed using relative humidity φ and temperature T as primitive driving potentials.

Let us consider first the transient term on the right hand side of the eq.(11), when the following expressions may be written

$$\frac{\partial W}{\partial t} = \frac{dW}{d\varphi} \frac{\partial \varphi}{\partial t} = \theta \frac{\partial \varphi}{\partial t}, \tag{12}$$

where $\theta = dW/d\varphi$ is the slope of the sorption isotherm $W = W(\varphi)$. The vapour pressure gradient can be transformed as

$$\vec{\nabla} p_v = \vec{\nabla} \left(p_s(T) \varphi \right) = \varphi \vec{\nabla} p_s + p_s \vec{\nabla} \varphi = \varphi \frac{dp_s}{dT} \vec{\nabla} T + p_s \vec{\nabla} \varphi. \tag{13}$$

Using Kelvin eq.(8) the suction pressure gradient in liquid conduction term can be treated as follows

$$\vec{\nabla} p_{suc}(T, \varphi) = \frac{\partial p_{suc}}{\partial T} \vec{\nabla} T + \frac{\partial p_{suc}}{\partial \varphi} \vec{\nabla} \varphi = -\frac{R \rho_l}{M_w} \left[\ln(\varphi) \vec{\nabla} T + \frac{T}{\varphi} \vec{\nabla} \varphi \right]. \tag{14}$$

Finally, substituting eqs.(12)-(14) into eq.(11) gives

$$\theta \frac{\partial \varphi}{\partial t} = \vec{\nabla} \cdot \left(D_\varphi \vec{\nabla} \varphi \right) + \vec{\nabla} \cdot \left(D_T \vec{\nabla} T \right) - \vec{\nabla} \cdot \left(D_l \rho_l \vec{g} \right), \tag{15}$$

where the primitive variable in eq.(15) is the relative humidity φ while the second and third term on the right side act as nonhomogeneous nonlinear source terms due to temperature gradient and gravity force. Notice that due to the second term the eq.(15) is coupled to the heat energy transport equation. The transport coefficients D_φ and D_T are given as:

$$D_\varphi = \delta_p p_s + D_l \frac{R \rho_l}{M_w} \frac{T}{\varphi} \qquad \text{and} \qquad D_T = \delta_p \varphi \frac{dp_s}{dT} + D_l \frac{R \rho_l}{M_w} \ln(\varphi). \tag{16}$$

If a moisture content W is used as a flow driving potential, using relations

$$W = W(\varphi) \qquad \text{and} \qquad \vec{\nabla} W = \frac{dW}{d\varphi}\vec{\nabla}\varphi = \theta\vec{\nabla}\varphi, \tag{17}$$

the eq.(15) can be rewritten as follows

$$\frac{\partial W}{\partial t} = \vec{\nabla}\cdot\left(D\vec{\nabla}W\right) + \vec{\nabla}\cdot\left(D_T\vec{\nabla}T\right) - \vec{\nabla}\cdot(D_l\rho_l\vec{g}) \qquad \text{and} \qquad D = D_\varphi/\theta. \tag{18}$$

2.2 Heat energy transport equation

The heat energy balance equation considers accumulation within the control volume, energy flux (sensitive and latent) in and out of the control volume and heat source/sink term [1, 2, 3], as follows

$$(c_{pm}\rho_m + c_{pl}W)\frac{\partial T}{\partial t} = -\vec{\nabla}\cdot\vec{q} = -\vec{\nabla}\cdot\vec{q}_{sens} - \vec{\nabla}\cdot\vec{q}_{lat} + I, \tag{19}$$

where the specific capacities c_{pm} and c_{pl} per mass refer to the dry porous material and to liquid water, respectively. The heat capacity of the air/water vapour mixture in the pores is neglected. The sensible heat energy flux $\vec{q}_{sens}$ can be given by Fourier model

$$\vec{q}_{sens} = -\lambda_{eff}(W)\vec{\nabla}T, \tag{20}$$

while for the latent heat flux $\vec{q}_{lat}$ one writes the following expression

$$\vec{q}_{lat} = \left[h_e + T(c_{pv} - c_{pl})\right]\vec{j}_v = -h_{lat}\delta_p\vec{\nabla}p_v, \tag{21}$$

where the quantities h_{lat}, h_e, c_{pv} and c_{pl} denote specific latent enthalpy, specific latent enthalpy of evaporation or condensation, specific heat of water vapour and specific heat of liquid water, respectively. Substituting heat flux eq.(20) and (21) into conservation eq.(19) results in the governing heat energy transport equation

$$c_{eff}\frac{\partial T}{\partial t} = \vec{\nabla}\cdot(\lambda_{eff}\vec{\nabla}T) + \vec{\nabla}\cdot(h_{lat}\delta_p\vec{\nabla}p_v) + I, \tag{22}$$

where coefficients c_{eff} and λ_{eff} are the effective specific heat per unit volume and effective thermal conductivity, respectively.

3 Boundary-domain integral equations

3.1 Integral representation of heat energy kinetics

Let us first consider the nonlinearity in unsteady energy transport eq.(22) caused by nonlinear transport properties, e.g. specific heat per volume $c_{eff} = c_{eff}(W,T)$ and heat conductivity $\lambda_{eff} = \lambda_{eff}(W,T)$ [6, 9]. They can be partitioned into a constant and a

variable part, e.g. $c_{eff} = c_o + \widetilde{c}$ and $\lambda_{eff} = \lambda_o + \widetilde{\lambda}$, enabling to partition the eq.(22) into a linear and nonlinear part in the following manner

$$\frac{\partial T}{\partial t} = a_o \frac{\partial^2 T}{\partial x_j \partial x_j} + \frac{1}{c_o}\left[\frac{\partial}{\partial x_j}\left(\widetilde{\lambda}\frac{\partial T}{\partial x_j} + h_{lat}\delta_p \frac{\partial p_v}{\partial x_j})\right) - \widetilde{c}\frac{\partial T}{\partial t} + I\right], \tag{23}$$

with $a_o = \lambda_o / c_o$. The integral representation of the nonlinear energy diffusion equation can now be derived considering the linear parabolic diffusion differential operator [6]

$$L\left[T\right] + b = a_o \frac{\partial^2 T}{\partial x_j \partial x_j} - \frac{\partial T}{\partial t} + b = 0, \tag{24}$$

with the corresponding integral equation written for a time step $\Delta t = t_F - t_{F-1}$

$$\begin{aligned} c\left(\xi\right) T\left(\xi, t_F\right) + a_o \int_\Gamma \int_{t_{F-1}}^{t_F} T q^\star dt d\Gamma = a_o \int_\Gamma \int_{t_{F-1}}^{t_F} q u^\star dt d\Gamma \\ + \int_\Omega \int_{t_{F-1}}^{t_F} b u^\star dt d\Omega + \int_\Omega T_{i,F-1} u^\star_{F-1} d\Omega, \end{aligned} \tag{25}$$

where $q = \partial T / \partial n = q_j n_j$ and $u^\star$ is the parabolic diffusion fundamental solution [8]. The domain integral of the pseudo-body forces includes the effects nonlinear transport properties, latent heat and the heat source term, namely

$$b = \frac{1}{c_o}\left[\frac{\partial}{\partial x_j}\left(\widetilde{\lambda} q_j + h_{lat}\delta_p \frac{\partial p_v}{\partial x_j}\right) - \widetilde{c}\frac{\partial T}{\partial t} + I\right] = \frac{1}{c_o}\left[\frac{\partial b_j}{\partial x_j} - \widetilde{c}\frac{\partial T}{\partial t} + I\right], \tag{26}$$

therefore the following integral representation can be obtained

$$\begin{aligned} c\left(\xi\right) T\left(\xi, t_F\right) + a_o \int_\Gamma \int_{t_{F-1}}^{t_F} T q^\star dt d\Gamma = a_o \int_\Gamma \int_{t_{F-1}}^{t_F} q u^\star dt d\Gamma + \frac{1}{c_o}\int_\Gamma \int_{t_{F-1}}^{t_F} b_j n_j u^\star dt d\Gamma \\ - \frac{1}{c_o}\int_\Omega \int_{t_{F-1}}^{t_F} b_j q_j^\star dt d\Omega + \frac{1}{c_o}\int_\Omega \int_{t_{F-1}}^{t_F} \left(-\widetilde{c}\frac{\partial T}{\partial t} + I\right) u^\star dt d\Omega + \int_\Omega T_{F-1} u^\star_{F-1} d\Omega. \end{aligned} \tag{27}$$

For the solution of eq.(27), the boundary Γ and domain Ω are discretized into a series of boundary elements and series of internal cells, respectively. Furthermore, functions and their derivatives are assumed to vary within each element or cell and each time step according to the space $\{\Phi\}$ or $\{\phi\}$ and time $\{\Psi\}$ interpolation functions such that

$$\begin{aligned} T(S,t) &= \{\Phi\}^T \{\Psi\} \{T\}^n_m, \qquad q(S,t) = \{\Phi\}^T \{\Psi\} \{q\}^n_m, \\ b_j(S,t) &= \{\Phi\}^T \{\Psi\} \{b_j\}^n_m, \qquad b_j(s,t) = \{\phi\}^T \{\Psi\} \{b_j\}^n_m, \quad \text{etc.}, \end{aligned} \tag{28}$$

where index n refers to the number of nodes within each element or cell, and the index m refers to the degree of variation of the function $\{\Psi\}$.

3.1.1 Linear time interpolation

Assuming linear variation of all functions within the individual time increment $\tau = t_F - t_{F-1}$, i.e. $m = 1, 2$ and

$$\Psi_1 = \frac{t_F - t}{\tau} \qquad \text{and} \qquad \Psi_2 = \frac{t - t_{F-1}}{\tau} \tag{29}$$

the analytical expressions for the time integrals can be derived, i.e. for the plane geometry

$$\begin{aligned}
U_1^\star &= a_o \int_{t_{F-1}}^{t_F} \Psi_1 u^\star dt = \frac{1}{4\pi}\left[\exp(-x_{F-1}) - x_{F-1}E_1(x_{F-1})\right], \\
U_2^\star &= a_o \int_{t_{F-1}}^{t_F} \Psi_2 u^\star dt = \frac{1}{4\pi}\left[E_1(x_{F-1}) - \exp(-x_{F-1}) + x_{F-1}E_1(x_{F-1})\right], \\
Q_1^\star &= a_o \int_{t_{F-1}}^{t_F} \Psi_1 q^\star dt = \frac{[x_i(\xi) - x_i(s)]n_i(s)}{8\pi a_o \tau} E_1(x_{F-1}), \\
Q_2^\star &= a_o \int_{t_{F-1}}^{t_F} \Psi_2 q^\star dt = \frac{[x_i(\xi) - x_i(s)]n_i(s)}{8\pi a_o \tau}\left[\frac{1}{x_{F-1}}\exp(-x_{F-1}) - E_1(x_{F-1})\right].
\end{aligned} \tag{30}$$

where E_1 is the exponential integral function and $x_{F-1} = r^2/4a_o\tau$; the eq. (27) can be rewritten as follows

$$\begin{aligned}
c\,(\xi)\,T_2\,(\xi) + \sum_{m=1}^{2}\sum_{e=1}^{E}\left[\int_{\Gamma_e}\{\Phi\}^T Q_m^\star d\Gamma\right]\{T\}_m^n = \sum_{m=1}^{2}\sum_{e=1}^{E}\left[\int_{\Gamma_e}\{\Phi\}^T U_m^\star d\Gamma\right]\{q\}_m^n \\
+ \sum_{m=1}^{2}\sum_{e=1}^{E}\left[\int_{\Gamma_e}\{\Phi\}^T U_m^\star n_j d\Gamma\right]\left\{\frac{b_j}{\lambda_o}\right\}_m^n - \sum_{m=1}^{2}\sum_{c=1}^{C}\left[\int_{\Omega_c}\{\phi\}^T Q_{jm}^\star d\Omega\right]\left\{\frac{b_j}{\lambda_o}\right\}_m^n \\
+ \sum_{c=1}^{C}\left[\int_{\Omega_c}\{\phi\}^T u_{F-1}^\star d\Omega\right]\{T\}_{F-1}^n.
\end{aligned} \tag{31}$$

In the discretized eq.(31) there are the following boundary and domain integrals, which are the functions of geometry, time step and material properties

$$\begin{aligned}
h_{em}^n = \int_{\Gamma_e}\{\Phi\}^T Q_m^\star d\Gamma, \quad g_{em}^n &= \int_{\Gamma_e}\{\Phi\}^T U_m^\star d\Gamma, \quad c_{ejm}^n = \int_{\Gamma_e}\{\Phi\}^T U_m^\star n_j d\Gamma, \\
d_{cjm} &= \int_{\Omega_c}\{\phi\}^T Q_{jm}^\star d\Omega, \quad b_c = \int_{\Omega_c}\{\phi\}^T u_{F-1}^\star d\Omega,
\end{aligned} \tag{32}$$

yielding the following discretized form of eq.(31)

$$\begin{aligned}
c\,(\xi)\,T_2\,(\xi) + \sum_{m=1}^{2}\sum_{e=1}^{E}\{h\}_m^T\{T\}_m^n &= \sum_{m=1}^{2}\sum_{e=1}^{E}\{g\}_m^T\{q\}_m^n + \sum_{m=1}^{2}\sum_{e=1}^{E}\{c_j\}_m^T\left\{\frac{b_j}{\lambda_o}\right\}_m^n \\
- \sum_{m=1}^{2}\sum_{c=1}^{C}\{d_j\}_m^T\left\{\frac{b_j}{\lambda_o}\right\}_m^n &+ \sum_{c=1}^{C}\{b\}^T\{T\}_1^n.
\end{aligned} \tag{33}$$

If the above statement is now applied, using the collocation method, to all boundary and domain nodes, and applying the notation, e.g. $[H] = [c(\xi)] + [\widehat{H}]$ and $[E_j] = [C_j] - [D_j]$, yielding a nonlinear system of equations

$$[H]_2\{T\}_2 + [H]_1\{T\}_1 = [G]_2\{q\}_2 + [G]_1\{q\}_1 + [E_j]_2\{\frac{b_j}{\lambda_o}\}_2 + [E_j]_1\{\frac{b_j}{\lambda_o}\}_1 + [B]\{T\}_1. \quad (34)$$

3.2 Integral representation of moisture kinetics

The integral representation of the moisture parabolic diffusion transport eq.(18) can be derived in a manner as it was obtained for the parabolic diffusion heat energy transport equation, e.g. using linear diffusion differential operator with $D = D_o + \widetilde{D}$, therefore eq.(18) may be reformulated as

$$L\left[W\right] + b = D_o \frac{\partial^2 W}{\partial x_j \partial x_j} - \frac{\partial W}{\partial t} + b = 0, \quad (35)$$

with the following integral representation

$$c\left(\xi\right) W\left(\xi, t_F\right) + D_o \int_\Gamma \int_{t_{F-1}}^{t_F} W q^\star dt d\Gamma = D_o \int_\Gamma \int_{t_{F-1}}^{t_F} \frac{\partial W}{\partial n} u^\star dt d\Gamma + \int_\Omega \int_{t_{F-1}}^{t_F} b u^\star dt d\Omega + \int_\Omega W_{i,F-1} u^\star_{F-1} d\Omega. \quad (36)$$

The domain integral incorporates two terms, e.g. the first one describes the mass flux due to nonlinear transport diffusivity while the second one describes mass flux due to temperature gradient

$$b = \frac{\partial}{\partial x_j}\left(\widetilde{D}\frac{\partial W}{\partial x_j} + D_T \frac{\partial T}{\partial x_j}\right) = \frac{\partial b_j}{\partial x_j}, \quad (37)$$

yielding the following integral representation

$$c\left(\xi\right) W\left(\xi, t_F\right) + D_o \int_\Gamma \int_{t_{F-1}}^{t_F} W q^\star dt d\Gamma = D_o \int_\Gamma \int_{t_{F-1}}^{t_F} q u^\star dt d\Gamma + \int_\Gamma \int_{t_{F-1}}^{t_F} b_j n_j u^\star dt d\Gamma - \int_\Omega \int_{t_{F-1}}^{t_F} b_j q_j^\star dt d\Omega + \int_\Omega W_{F-1} u^\star_{F-1} d\Omega. \quad (38)$$

3.2.1 Linear time interpolation

Assuming linear variation of all field functions within the individual time increment, the eq. (38) is written as follows

$$c\left(\xi\right) W_2\left(\xi\right) + \sum_{m=1}^{2}\sum_{e=1}^{E}\{h\}_m^T\{W\}_m^n = \sum_{m=1}^{2}\sum_{e=1}^{E}\{g\}_m^T\{q\}_m^n + \sum_{m=1}^{2}\sum_{e=1}^{E}\{c_j\}_m^T\left\{\frac{b_j}{D_o}\right\}_m^n - \sum_{m=1}^{2}\sum_{c=1}^{C}\{d_j\}_m^T\left\{\frac{b_j}{D_o}\right\}_m^n + \sum_{c=1}^{C}\{b\}^T\{W\}_1^n. \quad (39)$$

Based on the development of the final discretized eq.(34) for the energy transport, on may state the following discretized representation of the moisture transport

$$[H]_2\{W\}_2 + [H]_1\{W\}_1 = [G]_2\{q\}_2 + [G]_1\{q\}_1 + [E_j]_2\{\frac{b_j}{D_o}\}_2 + [E_j]_1\{\frac{b_j}{D_o}\}_1 + [B]\{W\}_1. \quad (40)$$

4 Numerical algorithm

To decrease storage and CPU time requirements of the single domain BEM we employ the macro element approach [5, 7]. The idea is to use a collocation scheme for integral equation for each domain cell separately and require that the field functions and their normal derivatives must obey some restriction conditions over the domain cell boundaries. Since every domain cell is neighbour only to a few cells we end up with a sparse system of equations. In a nutshell, we are using single domain BEM on every domain cell separately and connect them via compatibility and equilibrium conditions. The heat energy transport equation and moisture transport equation represent coupled nonlinear system of equations which can only be solved iteratively.

5 Numerical example - moisture uptake in a semi-infinite region

The benchmark test example, shown in fig.(1), deals with a $L = 6.0m$ or $L = 18.0m$ long and $H = 2.0m$ width single homogeneous material in equlibrium with a constant surrounding environment [3]. The material is perfectly air tight. At a certain moment the temperature and the relative humidity undergoes a step change.

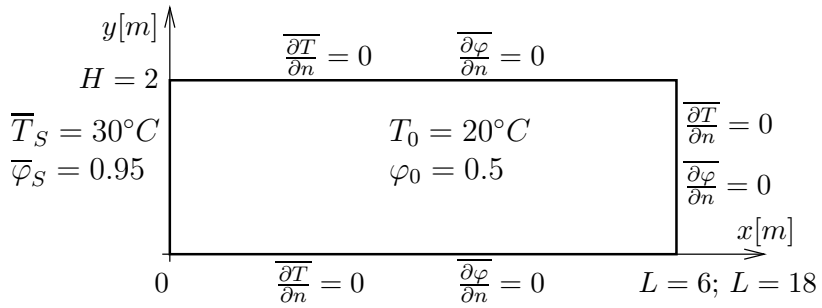

Figure 1: A semi-infinite homogeneous structure: geometry, boundary and initial conditions

One non-uniform non-symmetric mesh of $M = 400 \times 2$ macro elements is used, with the ratio $R_x = 10$ between the largest and the smallest boundary element. Convergence criterion was selected as $\epsilon = 10^{-7}$. The time-dependent analysis was performed by running the simulation from the initial state with a time step value of $\triangle t = 1.0$. The objective is to calculate the moisture and temperature distribution after $t = 7, 30$ and 365 days. The short solution domain, e.g. $L = 6.0m$, is used to simulate time dependent behaviour for 7 and 30 days, while the long solution domain, e.g. $L = 18.0m$ is applied for the 365

days simulation. The initial hygrothermal conditions of the structure are temperature $T_o = 20^oC$ and relative humidity $\varphi_o = 0.50$. After the step change the left surface of the structure is exposed to $\varphi_S = 0.95$ relative humidity and temperature $T_S = 30^oC$, respectively, while on the right surface the normal derivatives of the corresponding field functions are prescribed to be zero. Therefore, the following boundary conditions of the first kind can be prescribed on the left boundary at $x = 0$

$$W_S = \overline{W(\varphi_S)} \quad \text{and} \quad T_s = \overline{T_S} \quad \text{on} \quad x = 0 \text{ and } 0 \leq y \leq H \text{ for } t > 0, \tag{41}$$

and on all other boundaries the zero boundary conditions of the second kind are prescribed

$$\frac{\partial W}{\partial n} = 0 \quad \text{and} \quad \frac{\partial T}{\partial n} = 0 \quad \text{for} \quad t > 0, \tag{42}$$

while the initial conditions are

$$\begin{aligned} W &= \overline{W_o(\varphi_o)} \quad \text{and} \quad T = \overline{T}_o \quad \text{in} \quad \Omega \quad \text{at} \quad t = 0, \\ \frac{\partial W}{\partial n} &= \overline{\frac{\partial W_o}{\partial n}} \quad \text{and} \quad \frac{\partial T}{\partial n} = \overline{\frac{\partial T_o}{\partial n}} \quad \text{on} \quad \Gamma \quad \text{at} \quad t = 0. \end{aligned} \tag{43}$$

The sorption isotherm is given by an expression [3]

$$W(\varphi) = \frac{146}{\left[1 + (-8.0 \cdot 10^{-8} R_w T \rho_l \ln(\varphi))^{1.6}\right]^{0.375}}. \tag{44}$$

The vapour diffusivity/permeability δ_p and liquid water diffusivity/conductivity D_l transport coefficients are given by the expressions [3]

$$\delta_p = \frac{M_w}{RT} \frac{26.1 \cdot 10^{-6}}{200} \frac{1 - \frac{W}{146}}{0.503(1 - \frac{W}{146})^2 + 0.497}, \tag{45}$$

$$\begin{aligned} D_l = \exp\big(&-39.2619 + 0.0704(W - 73) - 1.7420 \cdot 10^{-4}(W - 73)^2 \\ &-2.7952 \cdot 10^{-6}(W - 73)^3 - 1.1566 \cdot 10^{-7}(W - 73)^4 + 2.5969 \cdot 10^{-9}(W - 73)^5\big). \end{aligned} \tag{46}$$

Effective thermal conductivity and specific heat per volume for dry material, respectively, are given by the relations [3]

$$\lambda_{eff} = 1.5 + \frac{15.8}{1000} W \qquad \text{and} \qquad c_{pm}\rho_m = c_m = 1.824 \cdot 10^6. \tag{47}$$

The numerical simulation results are shown in fig.(2) and fig.(3) for the moisture and temperature distributions, respectively, for a time increment $\triangle t = 1.0$ day for the moisture and temperature distributions in 7 , 30 and 365 days. The numerical results obtained are in excellent agreement with a bench mark results given with the limit values [3] which are marked with $\circ, \square, \triangledown$ in the figures.

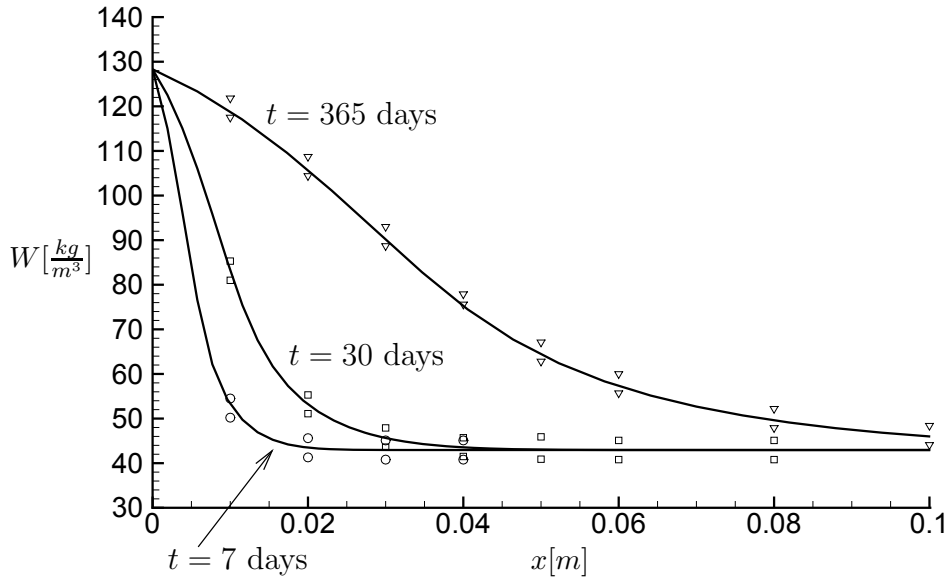

Figure 2: The moisture distribution at 7, 30 and 365 days: mesh $M = 400 \times 2$; time step $\triangle t = 1.0$ day; $\circ, \square, \triangledown$ - limits of validity for numerical results [3]

6 Conclusions

The boundary element method has been formulated and implemented to solve two-dimensional time dependent coupled nonlinear heat and moisture transfer through porous solid. Quadratic basis functions to approximate field functions and constant interpolation for fluxes are used, while linear variations of all functions over each individual time step is considered. The test bench mark example is highly nonlinear coupled moisture and heat transport, numerical solution of which is severe due to very different time and length scales caused by the difference of several order in the magnitude of heat and mass diffusion coefficients. The linear variation of functions over time step model proved to be very accurate and stabile numerical model.

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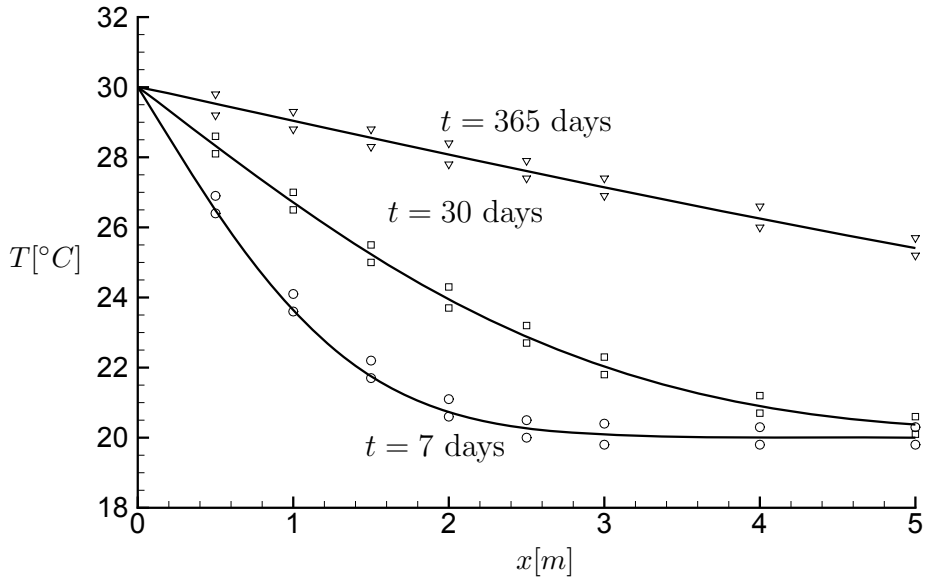

Figure 3: The temperature distribution at 7, 30 and 365 days: mesh $M = 400 \times 2$; time step $\triangle t = 1.0$ day; $\circ, \square, \triangledown$ - limits of validity for numerical results [3]

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